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TECHNICAL NOTES

Unsteady thermal field in a long, prismatic rod with a complicated initial condition and adiabatic boundary

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INTRODUCTION

CONSIDER the thermal problem shown in Fig. 1 governed by the differential system

$$k\nabla^2 T = \rho \cdot c_p \cdot \frac{\partial T}{\partial t} \tag{1}$$

$$T(x,y,0) = T_0 \quad \text{if} \quad (x,y) \in C_0 \tag{2a}$$

$$T(x,y,0) = 0 \quad \text{if} \quad (x,y) \in D_0 - C_0 \tag{2b}$$

$$\frac{\partial T}{\partial n}[L(x,y) = 0] = 0 \tag{3}$$

where $L(x,y) = 0$ is the functional relation which describes the boundary of the domain and n is the outer normal to the boundary.

Using the classical method of separation of variables one makes

$$T(x,y,t) = T_1(x,y)\tau(t). \tag{4}$$

Substituting (4) in (1) one obtains

$$\nabla^2 T_1 + \beta^2 T_1 = 0 \tag{5a}$$

$$\tau'(t) + \frac{k}{c_p \rho}\beta^2 \tau(t) = 0 \tag{5b}$$

where β^2 is the separation constant.

In the case of a domain with s-axes of symmetry the analytic function which transforms the given domain in the z-plane onto a unit circle in the ξ-plane is given by [1]

$$z = f(\xi) = a_p A_s \sum_0^\infty (-1)^n a_n \xi^{1+sn}; \quad \xi = r\,e^{i\theta} \tag{6}$$

where

$$a_0 = 1; \quad a_n = a_{n-1}\frac{[(n-1)n+1][(n-1)s+2]}{ns(ns+1)}$$

$$A_s = \text{constant [1]}; \quad a_p = \text{apothem}.$$

Consider now a circle of radius $R_0 \ll a_p$ (see Fig. 1). Obviously, when $|\xi| \ll 1$ the first term of the infinite series (6) predominates and one has

$$R_0 \cong a_p A_s r_0$$

and then

$$r_0 \cong R_0/a_p A_s. \tag{7}$$

Substituting (6) in (5a) results in the transformed partial differential equation

$$\nabla^2 T_1 + \beta^2 |f'(\xi)|^2 T_1 = 0 \tag{8}$$

while the governing boundary condition is:

$$\left.\frac{\partial T_1}{\partial r}\right|_{r=1} = 0. \tag{9}$$

It is important to point out that conformal mapping techniques for solving two dimensional transient heat conduction problems have been used by other researchers [2].

As a first approximation one expresses the approximate solution of (8) in the form

$$T_1 \cong T_{1a} = \sum B_m J_0(\alpha_m r) \tag{10}$$

where, in order to satisfy (9)

$$\left.\frac{\mathrm{d}J_0(\alpha_m r)}{\mathrm{d}r}\right|_{r=1} = 0.$$

Accordingly

$$J_1(\alpha_m) = 0. \tag{11}$$

The separation constants (β_m^2) can be calculated, in general, using one of the usual weighted residuals techniques. For the domains considered in this study they have been obtained from Ref. [3].

For $t = 0$ one must satisfy the initial conditions (2). This is now a straightforward matter in the ξ-plane since in view of (10) and (11) one writes

$$T_{1a}(r,t)|_{t=0} = B_0 + \sum_{m=1}^{M} B_m J_0(\alpha_m r). \tag{12}$$

Accordingly

$$B_0 = T_0 r_0^2; \quad B_m = \frac{2r_0}{\alpha_m} T_0 \frac{J_1(\alpha_m r_0)}{J_0^2(\alpha_m)}. \tag{13}$$

In conclusion the approximate solution of the thermal problem under study may be expressed, in the ξ-plane, as

$$T \cong B_0 + \sum_{m=1}^{M} B_m J_0(\alpha_m r)\exp\left(-\frac{k}{c_p \rho}\beta_m^2 t\right). \tag{14}$$

NUMERICAL RESULTS

From ref. [2] one obtains the first two eigenvalues:

(a) square shape ($s = 4$)

$$(\beta_0 a_p)^2 = 0; \quad 3.32$$

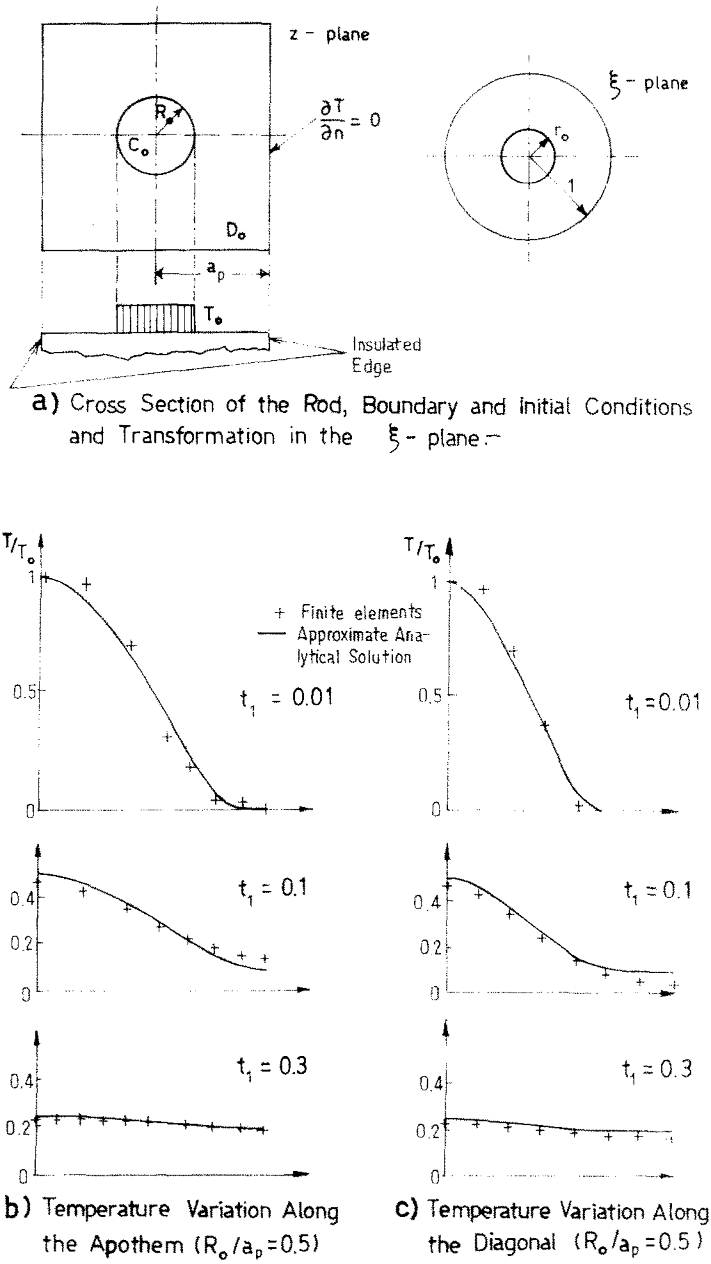

FIG. 1. Thermal system under study: square cross section.

(b) hexagonal shape ($s = 6$)

$$(\beta_0 a_p)^2 = 0; \quad 3.65.$$

Figures 1 and 2 depict dimension less temperature variations T/T_0 as a function of $t_1 = (k/c_p\rho)(t/a_p^2)$ along the apothem and the diagonal of square and hexagonal cross sections respectively.

The agreement with the results obtained by means of the finite element method is good as long as t_1 is sufficiently large as to require only two coordinate functions ($M = 1$ in equation (14) and the parameter R_0/a_p is small ($R_0/a_p \leqslant 0.5$). In fact it is quite remarkable the fact that a two term approximate expression is able to represent with reasonable engineering accuracy the thermal field in a previously unresolved situation where two types of complications arise: geometry of the domain (e.g. hexagonal shape) and complex definition of the initial condition.

Obviously the accuracy of the results can be improved taking into account the θ-variation in the ξ-plane and additional coordinate functions.

It is important to point out that the methodology presented herewith is quite simple and straightforward provided that the mapping function is known in advance. The limitation of the present technique is that it cannot handle irregular boundary value and boundary shape such as the method proposed in [2] which is considerably more complex but more general in scope.

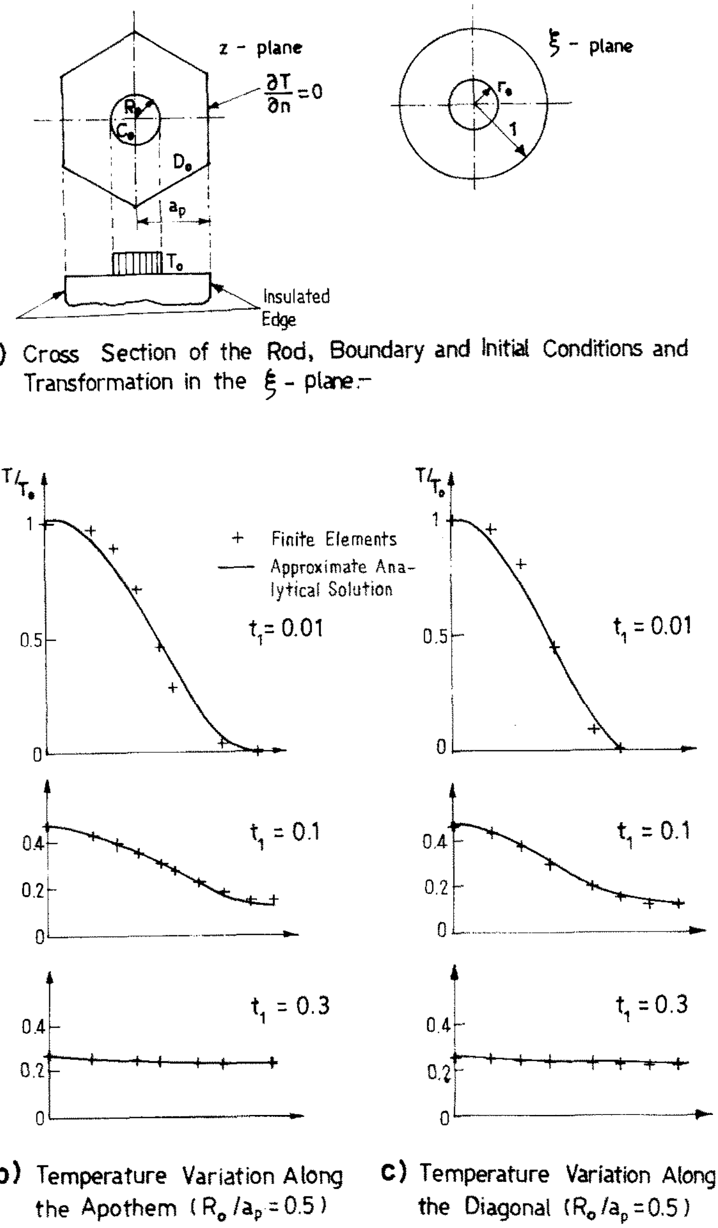

FIG. 2. Thermal system under study: hexagonal domain.

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